# Code : 031815

## B.Tech 8th Semester Exam., 2020

## MODERN CONTROL THEORY

*Time* : 3 hours *Full Marks* : 70

*Instructions* :

(i) *There are **NINE** questions in this paper.*

(ii) *Attempt **FIVE** questions in all.*

(iii) *Question No. **1** is compulsory.*

(iv) *The marks are indicated in the right-hand margin.*

1. Write brief explanation of any *seven* of the following : 2×7=14

(a) Similarity transformation

(b) Eigenvalues and eigenvectors

(c) Deadzone

(d) Negative-definiteness and positive semi-definiteness

(e) Jordan canonical form

(f) Reduced order observer

(g) Nilpotent matrix









(h) Quadratic performance index

(i) Saturation

(j) Observer design

2. (a) Find eigenvalues, eigenvectors and modal matrix for 5

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & -2 & 1 \\ 8 & 2 & -5 \end{bmatrix}$$

(b) Find the state transition matrix for the following system : 5

$$\dot{x} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} u; \; y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

(c) Show invariance of eigenvalues under similarity transformation. 4

3. (a) Consider the following system :

$$\dot{x} = \begin{bmatrix} 5 & 3 & 0 \\ 6 & -3 & 2 \\ 0 & 0 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \\ -6 \end{bmatrix} u; \; y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

(i) Find eigenvalues of $A$ and determine the stability of the system.

*(ii)* Find the transfer function model and from there, determine the stability of the system.

*(iii)* Are the two results same? If not, why? 7

*(b)* Give the Laplace transform approach to the solution of homogeneous state equations and construct the controllable canonical state-space representation for the system

$$\frac{50}{(s+4)^2(s+2)}$$ 7

**4.** *(a)* Consider the system given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-1 & -2 & -2\\ 0 & -1 & 1\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}2\\ 0\\ 1\end{bmatrix}u;\ y=[1\ \ 1\ \ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Is the system completely state controllable and completely observable? Explain how to obtain a transformation matrix in similarity transformation. 7

*(b)* Discuss precisely different types of stability of non-linear system and explain phase plane and phase trajectory with neat sketch. 7







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**5.** *(a)* Consider the system defined by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1{\cdot}688 & 0{\cdot}4537 & -5{\cdot}135\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1{\cdot}354\end{bmatrix}u;$$

$$y=[1\ \ 0\ \ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Given the set of desired poles for the observer to be $s=-2\pm 3\sqrt{3}j$ and $s=-7$. Design a full-order observer. 7

*(b)* Given the system of equations

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}2 & 1 & 5\\ 0 & 4 & 6\\ 2 & 3 & 11\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0{\cdot}5\\ 0\\ 0\end{bmatrix}u,\ \begin{bmatrix}x_1(0)\\ x_2(0)\\ x_3(0)\end{bmatrix}=\begin{bmatrix}1\\ -1\\ 0\end{bmatrix}$$

$$y=[1\ \ 1\ \ 1]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

where $u(t)$ is the unit-step input occurring at $t=0$.

Obtain the response $y(t)$ of the given system. 7

6. (a) A regulator system has a plant

$$\frac{Y(s)}{U(s)} = \frac{14}{(s+1)(s+4)(s+3)}$$

Define state variable as

$$x_1 = y, x_2 = \dot{x}_1, x_3 = \dot{x}_2$$

By use of the state feedback control $u = -Kx$, it is desired to place the closed-loop poles at $s = -2 \pm 2\sqrt{3}j$ and $s = -10$. Determine the necessary state-feedback gain matrix $K$. 7

(b) Consider the system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u; \quad y = [1 \quad 1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Transform the system of equations into—

(i) controllable canonical form,

(ii) observable cannonical form. 7

7. (a) Consider the system described by the equations 

$$x_1(k+1) = 3x_1(k) + 0{\cdot}45x_2(k) - 6$$

$$x_2(k+1) = 0{\cdot}7x_2(k) + 4$$

Investigate the stability of the equilibrium state. Use the direct method of Lyapunov. 7













(b) What is limit cycle? Discuss about the theorems by which the existence of limit cycle can be predicted. 7

8. (a) Consider the following :

$$\dot{x} = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \quad y = \begin{bmatrix} 1 & 0{\cdot}5 \\ 0 & 2 \end{bmatrix} x$$

Find the optimal control law that minimizes

$$J = \frac{1}{2}\int_0^\infty (y_1^2 + y_2^2 + 2u^2)dt$$ 7

(b) Explain singular points. Find out the singular points for the following systems :

(i) $y'' + 3y' - 10 = 0$

(ii) $y'' + 3y' + 2y = 0$

Show the trajectories of the singular points. 7

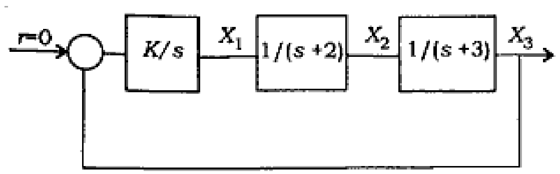
9. (a) Explain the following with an example each : 7

(i) Lyapunov main stability theorem

(ii) Lyapunov second method

(iii) Krasovskii theorem

*(b)* Using Lyapunov's direct method, find the range of $K$ to guarantee stability of the system shown in the figure below. Use the Lyapunov's direct method and Routh criterion and compare the results : 7

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