

B.Tech. 4th Semester Exam., 2014

DISCRETE MATHEMATICAL STRUCTURE
AND GRAPH THEORY

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
 (ii) There are **NINE** questions in this paper.
 (iii) Attempt **FIVE** questions in all.
 (iv) Question No. 1 is compulsory.

1. Choose the correct alternative from the following (any seven) : $2 \times 7 = 14$

(a) Consider a simple connected graph G with n vertices and n edges ($n > 2$). Then which of the following statements is True?

- (i) G has at least one cycle
 (ii) G has no cycles
 (iii) The graph obtained by removing any edge from G is not connected
 (iv) G has at least one cycle and the graph obtained by removing any edge from G is not connected

(b) The number of distinct simple graphs with up to three nodes is

(i) 9

(ii) 7

(iii) 10

(iv) 15

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(c) Consider the graph G , where $V(G) = \{A, B, C, D\}$ and $E(G) = \{\{A, B\}, \{B, C\}, \{C, D\}\}$. The degree of each of the vertices A, B, C and D respectively in G are

(i) 1, 2, 3, 2

(ii) 1, 3, 2, 2

(iii) 1, 1, 1, 1

(iv) 1, 2, 2, 1

(d) Let f and g be the functions defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$, then the composition of f and g is akubihar.com

(i) $6x + 6$ (ii) $5x + 5$ (iii) $6x + 7$ (iv) $7x + 5$

(e) Among 200 people, 150 either swim or jog, or both. If 85 swim, and 60 swim and jog, how many jog? akubihar.com

(i) 125

(ii) 225

(iii) 85

(iv) 25

(f) A graph in which all nodes are of equal degree is known as — graph.

(i) complete

(ii) multi

(iii) non-regular

(iv) regular

(g) The minimum number of spanning trees in a connected graph with n nodes is

(i) $n-1$

(ii) $n/2$

(iii) 2

(iv) 1

(h) The negation of 'Today is Friday' is

(i) Today is Saturday akubihar.com

(ii) Today is not Friday

(iii) Today is Thursday

(iv) Today is Sunday

(i) Whether the relation R on the set of all integers is reflexive, symmetric, anti-symmetric or transitive, where $(x, y) \in R$ if and only if $xy \geq 1$?

(i) Anti-symmetric

(ii) Transitive

(iii) Symmetric

(iv) Both symmetric and transitive

(j) If $p =$ 'It is raining' and $q =$ 'She will go to college', then 'It is raining and she will not go to college' will be denoted by

(i) $p \wedge \neg q$

(ii) $p \wedge q$

(iii) $\neg p \wedge q$

(iv) $\neg(p \wedge q)$

2. (a) Define the following terms and give an example for each :

Reflexive, Irreflexive, Anti-symmetric, Transitive, Partition set

(b) If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$ and $A_3 = \{5\}$, define relation R on A by xRy , if x and y are in the same subset A_i , for $1 \leq i \leq 3$. Is R an equivalence relation?

10+4=14

3. (a) Let $f, g: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, where $\forall x \in \mathbb{Z}^+$, $f(x) = x+1$ and $g(x) = \max\{1, x-1\}$, the maximum of 1 and $x-1$.
- (i) What is the range of f ?
 - (ii) Is f an onto function?
 - (iii) Is the function one-to-one?
 - (iv) What is the range of g ?
- (b) Let $A = \{1, 2, 3, 4\}$. Let R be a relation on A defined by xRy iff x/y and $y = 2x$.
- (i) Whether R is a relation of set of ordered pairs?
 - (ii) Draw digraph of R .
 - (iii) Determine in-degrees and out-degrees of a digraph. 8+6=14
4. (a) Define the following with example :
Group, Subgroup, Homomorphism,
Cyclic group, Coset
- (b) Determine whether f is one-one or onto for the following cases :
- (i) Let $A = B = \{1, 2, 3, 4\}$ and $f = \{(1, 1)(2, 3)(3, 4)(4, 2)\}$
 - (ii) Let $A = \{abc\}$, $B = \{1, 2, 3, 4\}$ and $f = \{(a, 1)(b, 1)(c, 4)\}$ 10+4=14



5. (a) State and prove De Morgan's laws of set theory. akubihar.com
- (b) In a survey of 260 college students, the following data were obtained :
- 64 had taken a mathematics course
 - 94 had taken a computer science course
 - 58 had taken a business course
 - 28 had taken both a mathematics and a business course
 - 26 had taken both a mathematics and a computer science course
 - 22 had taken both a computer science and a business course
 - 14 had taken all the three types of course. akubihar.com
- (i) How many of these students had taken none of the three courses?
 - (ii) How many had taken only a computer science course? 4+10=14
6. (a) Prove that for any non-empty binary tree T , if n_0 is the number of leaves and n_2 be the number of nodes having degree two, then $n_0 = n_2 + 1$.
- (b) Derive total number of nodes of a binary tree having depth n . akubihar.com 14

7. (a) Define 'group', 'order of a group', and 'Abelian group'.

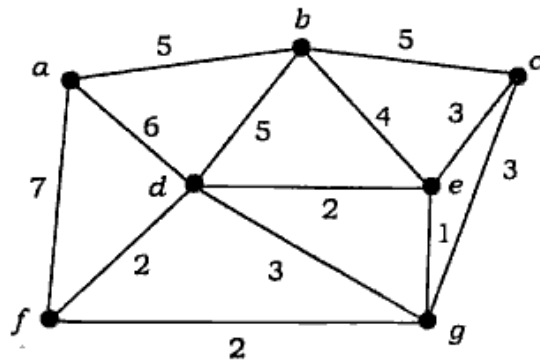
(b) For $P = \{p_1, p_2, \dots, p_5\}$ and $Q = \{q_1, q_2, \dots, q_5\}$, explain why $(P, *)$ and (Q, Δ) are not groups. The operations $*$ and Δ are given in the following table :

*	p_1	p_2	p_3	p_4	p_5	Δ	q_1	q_2	q_3	q_4	q_5
p_1	p_1	p_2	p_3	p_4	p_5	q_1	q_4	q_1	q_5	q_3	q_2
p_2	p_2	p_1	p_4	p_5	p_3	q_2	q_3	q_5	q_2	q_1	q_4
p_3	p_3	p_5	p_1	p_2	p_4	q_3	q_1	q_2	q_3	q_4	q_5
p_4	p_4	p_3	p_5	p_1	p_2	q_4	q_2	q_4	q_1	q_5	q_3
p_5	p_5	p_4	p_2	p_3	p_4	q_5	q_5	q_3	q_4	q_2	q_1

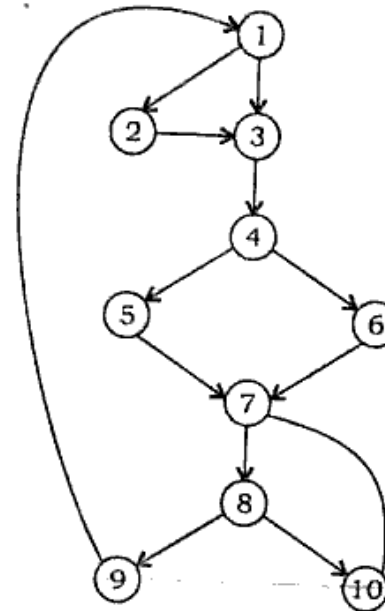
6+8=14

8. Explain Dijkstra's algorithm and apply it to the weighted graph $G = (V, E)$ shown in Figure below and determine the shortest distance from vertex a to each of the other vertices in the graph. akubihar.com

6+8=14



9. Consider the graph given below :



(a) Find the adjacency list and BFS traversal of the above graph.

(b) Prove that the maximum number of edges possible in a simple graph of n nodes is $n(n-1)/2$.

8+6=14