Code : 211303

B.Tech. 3rd Semester Exam., 2013

MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The questions are of equal value.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt any **FIVE** questions.*

(iv) Question No. 1 is compulsory.

1. Choose the correct answer of any *seven* out of *ten* :

(a) The value of integral $\int_k^{2-i} z\,dz$ is

(i) 0

(ii) 1

(iii) $1+2i$

(iv) $1-2i$

(b) The value of complex integral $\int_c \frac{z}{z^2+1}\,dz$, where c is a closed curve $|z+i|=0.5$, is

(i) πi

(ii) 0

(iii) $2\pi i$

(iv) $-\pi i$

(c) The value of $J_2(x)$ in terms of $J_1(x)$ and $J_0(x)$ is

(i) $2J_1(x) - xJ_0(x)$

(ii) $\frac{4}{x}J_1(x) - J_0(x)$

(iii) $\frac{2}{x}J_1(x) - \frac{2}{x}J_0(x)$

(iv) $\frac{2}{x}J_1(x) - J_0(x)$

(d) If $\int_{-1}^{1} P_n(x)dx = 2$, then n is

(i) 1

(ii) 0

(iii) -1

(iv) None of the above

(e) In the functions $Q_1(x) = (x-a)P_1(x)$ and $Q_2(x) = (x-a)^2 P_2(x)$, if Q_1 and Q_2 are analytic, thus $x = a$ is called

(i) ordinary singular point

(ii) irregular singular point

(iii) regular singular point

(iv) None of the above

(f) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(i) $z = f_1(y+x) + f_2(y-x)$

(ii) $z = f_2(y+x) + f_1(y-x)$

(iii) $z = f_2(y+x) + f_2(y-x)$

(iv) $z = f(x^2 - y^2)$

(g) The solution of $3x\frac{\partial z}{\partial x} - 5y\frac{\partial z}{\partial y} = 0$ is

(i) $f(x^3y, z^5) = 0$

(ii) $f(x^3y^3, z) = 0$

(iii) $f(xy, z) = 0$

(iv) $f(x^5y^3, z) = 0$

(h) The solution of $z = p + q$ is

(i) $f(x+y, y+\log_e z) = 0$

(ii) $f(x \cdot y, y\log_e z) = 0$

(iii) $f(x-y, y-\log_e z) = 0$

(iv) None of the above

(i) An unbiased coin is tossed 3 times. The probability of obtaining two heads is .

(i) $\frac{1}{2}$

(ii) $\frac{3}{8}$

(iii) 1

(iv) $\frac{1}{8}$

(j) The probability that a marksman will hit a target is given as $\frac{1}{5}$. Then his probability of at least one hit in 10 shots is

(i) $1 - \left(\frac{4}{5}\right)^{10}$

(ii) $\frac{1}{5^{10}}$

(iii) $1 - \frac{1}{5^{10}}$

(iv) None of the above

2. (a) Solve by Frobenius method, the differential equation $xy'' + y' + x^2y = 0$. Indicate the form of second solution which is linearly independent of the first obtained above.

(b) Prove :

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

3. (a) Prove :

(i) $nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$

(ii) $(x^2-1)P_n = (n+1)(P_{n+1} - xP_n)$

(b) Show :

$$\int_{-1}^{1} x^3 \cdot P_3(x)dx = \frac{4}{35}$$

4. (a) Form the partial differential equation from $2z = (ax+y)^2 + b$.

(b) Solve :

(i) $y^2 p - xyq = x(z-2y)$

(ii) $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ for $z(1, y) = \cos y$

5. (a) By separation of the variables, solve $\frac{\partial u}{\partial x} + u = \frac{\partial u}{\partial t}$, if $u = 4e^{-3x}$ for $t = 0$.

(b) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

under the condition $u = 0$, when $x = 0$ and $x = \pi$; $\frac{\partial u}{\partial t} = 0$, when $t = 0$ and $u(x, 0) = x$, $0 < x < \pi$.

6. (a) What are the sufficient conditions for a function $f(z)$ to be analytic? Test the analyticity of $\frac{1}{(z-1)(z+1)}$.

(b) Prove that $u = x^2 - y^2$ and

$$v = \frac{y}{x^2 + y^2}$$

are harmonic functions of $f(x, y)$ but are not harmonic conjugate.

7. (a) Evaluate

$$\int_C \frac{e^{2z}}{(z+1)^4} dz$$

where $|z-1| = 2$ is a circle.

(b) For the function $f(z) = \frac{4z-1}{z^4-1} + \frac{1}{z-1}$, find all Taylor or Laurent series about the centre zero.

8. (a) The frequency distribution of measurable characteristic varying between 0 and 2 is as under

$$f(x) = x^3, \quad 0 \le x \le 1$$
$$= (2-x)^3, \quad 1 \le x \le 2$$

Calculate the standard deviation and mean deviation about the mean.

(b) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0·05 :

x :	0	1	2	3	4
f :	419	352	154	56	19

Given, at 3 degree of freedom, $\chi^2_{0\cdot05} = 7\cdot82$.

9. (a) A die is thrown 8 times and it is required to find the probability that 3 will show—

(i) exactly 2 times;

(ii) at least 7 times;

(iii) at least once.

(b) Define probability density function. A function $f(x)$ is defined as

$$f(x) = \begin{bmatrix} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0, & x > 4 \end{bmatrix}$$

Show that it is a probability density function.

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