Code : 211202

B.Tech. 2nd Semester Exam., 2014

MATHEMATICS—II

Time : 3 hours

Full Marks : 70

Instructions :

(i) ***All*** *questions carry equal marks.*

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct or best alternatives of the following (any seven) :

(a) The series $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$ is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) None of the above

(b) The series whose *n*th term is $\sqrt{n^3+1}-\sqrt{n^3}$, is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) None of the above

(c) Which one of the following functions is not periodic?

(i) $f(x)=\cos 2x+\cos 3x+\cos 5x$

(ii) $f(x)=e^{i8\pi x}$

(iii) $f(x)=e^{(-7x)}\sin 10\pi x$

(iv) $f(x)=\cos 2x\cdot\cos 4x$

(d) The period of a constant function is

(i) defined

(ii) defined under conditions

(iii) not defined

(iv) None of the above

(e) $\int_0^\infty \frac{\sin t}{t}dt$ is equal to

(i) π

(ii) $\pi/2$

(iii) $\pi/4$

(iv) $\pi/3$

(f) Inverse Laplace transform of $\frac{e^{-3s}}{(s-2)^4}$ is

(i) $\begin{matrix} 1 & t<3 \\ \frac{1}{5}\frac{(t-3)^3}{4} & t>3 \end{matrix}$

(ii) $\begin{matrix} 0 & t<3 \\ \frac{1}{6}\frac{t^3}{6}e^2 & t>3 \end{matrix}$

(iii) $\begin{matrix} 0 & t<3 \\ 0 & t>3 \end{matrix}$

(iv) $\begin{matrix} 0 & t<3 \\ \frac{1}{6}\frac{(t-3)^3}{6}e^{2(t-3)} & t>3 \end{matrix}$

(g) The area of region bounded by the curve $\frac{y+8}{x} = x-2$ and the x-axis is

(i) 54

(ii) 36

(iii) 18

(iv) 12

(h) The value of $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$ is

(i) $\frac{4}{35}$

(ii) $\frac{3}{35}$

(iii) $\frac{8}{35}$

(iv) $\frac{6}{35}$

(i) $\nabla \times (\nabla \times \vec{A})$, where $\vec{A}$ is a vector, is equal to

(i) $\vec{A} \times \nabla \times \vec{A} - \nabla^2 \vec{A}$

(ii) $\nabla^2 \vec{A} + \nabla(\nabla \cdot \vec{A})$

(iii) $\nabla^2 \vec{A} + \nabla \times \vec{A}$

(iv) $\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(j) Stokes' theorem connects

(i) a line integral and a surface line integral

(ii) a surface integral and a volume integral

(iii) a line integral and a volume integral

(iv) gradient of a function and its surface integral

2. (a) Discuss the nature of convergency of an infinite geometric series.

(b) Test the convergence for the series

$$\frac{x}{1 \cdot 2} + \frac{x^2}{3 \cdot 4} + \frac{x^3}{5 \cdot 6} + \frac{x^4}{7 \cdot 8} + \cdots \text{ to } \infty$$

3. (a) Find the Laplace $(1 + \cos 2t)$ for the first principle.

(b) Find the Laplace transform of

$$\frac{(1-\cos t)}{t^2}$$

4. (a) Find the Laplace inverse of

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)}$$

(b) Find the Fourier series representing the function $f(x) = x$, $0 < x < 2\pi$ and sketch its graph from $x = -4\pi$ to $x = 4\pi$.

5. (a) Find the Fourier series of the function defined as

$$f(x) = \begin{cases} x+\pi & \text{for } 0 \le x \le \pi \\ -x-\pi & \text{for } -\pi \le x < 0 \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x)$$

(b) Expand $f(x) = e^x$ in a cosine series over $(0, 1)$.

6. *(a)* Evaluate :

$$\int_0^\infty \int_0^\infty \frac{e^{-x^3}}{\sqrt{x}} y^4 e^{-y^6} \, dx \, dy$$

(b) Evaluate $\iint_R xy \, dx \, dy$, where R is the quadrant of the circle $x^2 + y^2 = a^2$ and $x \geq 0$, $y \geq 0$.

7. *(a)* Find by double integration the area enclosed by the pair of curves $y = 2 - x$ and $y^2 = 2(2 - x)$.

(b) Compute $\iiint \frac{dx dy dz}{(x + y + z + 1)^3}$ if the region of integration is bounded by the coordinates plane and plane $x + y + z = 1$.

8. *(a)* A particle moves along a plane curve such that its linear velocity is perpendicular to the radius vector. Show that the path of the particle is a circle.

(b) Show that

$$\nabla\left(\frac{\vec{a} \cdot \vec{r}}{r^n}\right) = \frac{\vec{a}}{r^n} - \frac{n(\vec{a} \cdot \vec{r})}{r^{n+2}}(\vec{r})$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$, $\vec{a}$ is a constant vector.

9. *(a)* If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that

$$\text{div}(\text{grad}\, r^n) = n(n+1) r^{n-2}$$

(b) Verify Green's theorem for $\oint_C (2xy dx - y^2 dy)$, where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$.

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