Code : 105217

2012 (A)

DISCRETE MATHEMATICS AND GRAPH THEORY

Time : 3 hours   Full Marks : 70

Instructions :

(i) *The marks are indicated in right-hand margin.*

(ii) *There are* **TEN** *questions in this paper.*

(iii) *Attempt any* **FIVE** *questions.*

1. (a) If G is k-critical, prove that $d = k - 1$.   7

   (b) Prove that every planar graph is 6-vertex colourable.   7

2. (a) Draw a complement of a graph. Draw K_c 3,4. If G is self-complementary, prove that $d = 0, 1 \pmod 4$.   7

   (b) Prove that a vertex v of a tree is a cut vertex if and only if $d(v) > 1$.   7

3. (a) If G has a Hamilton path, prove that $W(G - S) = |S| + 1$ for every proper subset S of V.   7

   (b) Prove that $a' = \beta$ for bipartite graphs.   7

12AK—250/220   (Turn Over)

---

4. (a) State and prove Schur's theorem.   7

   (b) If G is 2-edge connected, prove that G has a disconnected orientation.   7

5. (a) With usual notation, prove that $t|K_n| = nn - 2$.   7

   (b) Explain the construction of $H_{m,n}$ and prove that it is m-connected.   7

6. (a) Let $T = \{\forall x\, p(x) \to p(f(x))\}$. Find a model of T. Show that T is not valid.   7

   (b) Let

   $$T = \{\forall x\, p(x,\, x),\ \forall x \forall y (p(x,\, y))\, \forall x\, \forall y\, \forall z\\ (p(x,\, y) \cap p(y,\, z) \to p(x,\, z))\}$$

   Give two distinct models of T. Briefly explain any connection between the models.   7

7. (a) Prove that sum of the degrees of the vertices of any finite graph is even.   7

   (b) Show that every simple graph has two vertices of the same degree.   7

8. (a) Show that a tree with n vertices has exactly $n - 1$ edges.   7

   (b) If u and v are two vertices of a tree, show that there is a unique path connecting them.   7

12AK—250/220   (Continued)

9. Show that any graph where the degree of every vertex is even has an Eulerian cycle. Show if there are exactly two vertices a and b of odd degree, then there is an Eulerian path from a to b. Show that if there are more than two vertices of odd degree, it is impossible to construct an Eulerian path.   14

10. For the following well-formed formula, give their meaning in English underproposed interpretation and state whether they are true or false. The interpretation I is defined as follows :

    The domain of discourse is the set of non-negative integers, p_1 is =, f is +, g_1 is $\times$, a_1 is 0 and b_1 is 1:   14

    (i) $\forall x \exists y (p(x, f(y, y)) \lor p(x, f(f(y, y), b)))$

    (ii) $\forall x \forall y (p(g(x, y)a) \to (p(x, a) \lor p(y, a)))$

    (iii) $\exists y (p(f(y, y), b))$

★ ★ ★