Code : 211101

2011

Time : 3 hours *Full Marks : 70*

Instructions :

(i) There are NINE questions in the paper. All questions carry equal marks.

(ii) Attempt FIVE questions in all.

(iii) Question No. 1 is compulsory.

1. Choose and write the correct answer (any seven) :

(a) The $(n+1)$th term in Maclaurin's series is

(i) $\frac{x^n}{n} f^n(a)$

(ii) $\frac{x^n}{\lfloor a} f^n(a)$

(iii) $\frac{1}{\lfloor n} f^n(0)$

(iv) $f^n(0)$

(b) The angle ϕ between the tangent and the radius vector is given by

(i) $\tan\phi = \frac{1}{r} \cdot \frac{d\theta}{dr}$

(ii) $\tan\phi = \frac{1}{r} \frac{dr}{d\theta}$

(iii) $\tan\phi = r \frac{dr}{d\theta}$

(iv) $\tan\phi = r \frac{d\theta}{dr}$

(c) The formula of L' Hospital's rule is

(i) $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \left[\frac{f(x)}{g(x)} \right]$

(ii) $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \left[\frac{f'(x)}{g'(x)} \right]$

(iii) $\lim_{x \to a} \frac{f(x)}{g(x)} = \left[\frac{f(a)}{g(a)} \right]$

(iv) $\lim_{x \to a} \frac{f(x)}{g(x)} = \left[\frac{f'(a)}{g'(a)} \right]$

(d) If $f(x, y) = c$, then $\frac{dy}{dx}$ is

(i) $\frac{\partial f}{\partial x}$

(ii) $\frac{\partial f}{\partial y}$

(iii) $-\frac{\partial f / \partial x}{\partial f / \partial y}$

(iv) $\frac{\partial f / \partial x}{\partial f / \partial y}$

(e) The order of differential equation whose general solution is given by

$$y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$$

where c_1, c_2, c_3, c_4, c_5 are arbitrary constants is

(i) 5

(ii) 4

(iii) 3

(iv) 2

(f) The solution of differential equation

$$y\frac{dy}{dx} = x - 1$$

satisfying $y(1) = 1$ is

(i) $y^2 = x^2 - 2x + 2$

(ii) $y^2 = 2x^2 - x - 1$

(iii) $y = x^2 - 2x + 2$

(iv) None of these

(g) If A is a symmetric metrics and $n \in N$, then A^n is

(i) symmetric

(ii) skew-symmetric

(iii) diagonal matrix

(iv) None of these

(h) If

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$$

the top row of A^{-1} is

(i) $[5 \quad 6 \quad 4]$

(ii) $[5 \quad -3 \quad 1]$

(iii) $[2 \quad 0 \quad -1]$

(iv) $[2 \quad -1 \quad \frac{1}{2}]$

(i) The sum of eigenvalues of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

is

(i) 5

(ii) 7

(iii) 9

(iv) 18

(j) The value of $\int_0^\infty \frac{t^2}{1+t^4}\,dt$ is

(i) $\frac{\pi}{\sqrt{2}}$

(ii) $\frac{\sqrt{\pi}}{2}$

(iii) $\frac{\pi}{2}$

(iv) $\frac{\pi}{4}$

2. (a) If $y = \cos(m \sin^{-1} x)$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

and hence find $y_n(0)$

(b) Determine

$$\lim\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$$

as $x \to 0$.

3. (a) If $u = \log_e(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

(b) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$

4. (a) If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again at (x_2, y_2), then prove that

$$\frac{x_2}{x_1} + \frac{y_2}{y_1} = 1$$

(b) Find the pedal equation of the curve

$$r^m = a^m \cos m\theta$$

5. (a) Use Gauss-Jordan reduction method to compute the inverse of the matrix

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

by applying elementary row transformation.

(b) Find the rank by elementary row transformation of

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

6. (a) Solve by the elimination method (Gauss-Jordan method) :

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

(b) The matrix A is defined as

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

Find the eigenvalues of

$$3A^3 + 5A^2 - 6A + 2I$$

7. (a) Solve :

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$$

(b) Solve :

$$x(x-1)\frac{dy}{dx} - (x-2)y = x^2(2x-1)$$

8. (a) Prove duplication formula

$$\Gamma m \, \Gamma\left(m + \tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$$

(b) Show that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right) \Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$$

9. (a) Find the point upon the plane $ax + by + cz = p$ at which the function has a minimum value and find this minimum value.

(b) Define error function and prove that $\text{erf}(\infty) = 1$.

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