

B.Tech 1st Semester Exam., 2018 (New)

MATHEMATICS—I

(Calculus and Linear Algebra)

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. 1 is compulsory.

1. Choose the correct answer (any seven) :

2×7=14

(a) If $\phi(x, y, z) = 3x^2y - y^3z^2$, then $\nabla\phi$ at the point (1, 1, -2) is

(i) $12i - 9j - 16k$

~~(ii) $6i - 9j + 4k$~~

(iii) $6i - 9j - 4k$

(iv) $-12i - 9j + 16k$

(b) If $a < b$, then $\int_a^b |(x-a) + (x-b)| dx$ is equal to

(i) $\frac{(b-a)^2}{2}$

(ii) $\frac{(b^2 - a^2)}{2}$

(iii) $\frac{(a^3 - b^3)}{2}$

(iv) $(b-a)^2$

(c) The value of $\iint x^2y^2 dx dy$ over the region $x^2 + y^2 \leq 1$ is

(i) $\frac{\pi}{6}$

(ii) $\frac{\pi}{12}$

(iii) $\frac{\pi}{24}$

(iv) $\frac{\pi}{48}$

(d) If $A(2) = 2i - j + 2k$, $A(3) = 4i - 2j + 3k$, then $\int_2^3 A \cdot \frac{dA}{dt} dt$ is

(i) 5

(ii) 10

(iii) 15

(iv) 20

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(e) The Fourier series of the periodic function $f(x) = x + x^2$, $-\pi < x \leq \pi$ at $x = \pi$ converges to

- (i) π (ii) 2π
 (iii) π^2 (iv) $\pi + \pi^2$

(f) If $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \dots, \lambda_n$ are the eigenvalues of the matrix A , then $\lambda_1 - 2, \lambda_2 - 2, \lambda_3 - 2, \lambda_4 - 2, \dots, \lambda_n - 2$ shall be the eigenvalues of the matrix

- (i) A (ii) $A - I$
 (iii) $A - 2I$ (iv) $A - 3I$

(g) If X is the eigenvector of matrix A , then which is not the eigenvector of A^{-1} ?

- (i) $-X$ (ii) X^{-1}
 (iii) X (iv) $2X$

(h) If λ is an eigenvalue of matrix A , then $\frac{1}{\lambda}$ is the eigenvalue of

- (i) A^{-2} (ii) A^{-1}
 (iii) A^2 (iv) A

(i) The maximum value of $f(x) = x^n(1-x)^n$ for a natural number $n \geq 1$ and $0 \leq x \leq 1$ is

- (i) $\frac{1}{4^n}$ (ii) $\frac{1}{3^n}$
 (iii) $\frac{1}{2^n}$ (iv) 1

(j) The following series

$$1 + \frac{1}{2^p} + \frac{1}{4^p} + \frac{1}{6^p} + \frac{1}{8^p} + \dots$$

is convergent for

- (i) $p < 1$ (ii) $p \geq 1$
 (iii) $p \leq 1$ (iv) $p > 1$

2. (19) Find the evolutes of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(20) Evaluate the integral $\int_0^{\infty} x^n e^{-k^2 x^2} dx$, if $n > -1$. 7+7

3. (21) Find the area of the surface formed by the revolution of $x^2 + 4y^2 = 16$ about its major axis.

(b) Evaluate the following integral :

$$\int_0^1 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy dx \quad 7+7$$

4. (a) If

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x+\theta h)$$

find the value of θ as $x \rightarrow a$, $f(x)$ being $(x-a)^{5/2}$.

(b) Test the convergence of the series

$$\left(\frac{1}{2 \cdot 4}\right)^p + \left(\frac{1 \cdot 3}{2 \cdot 4 \cdot 6}\right)^p + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8}\right)^p + \dots \quad 7+7$$

5. (a) Show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at $(0, 0)$, but partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ both exist and have value 0.

(b) Show that the right circular cylinder of given surface (including the ends) and maximum volume is such that its height is equal to the diameter of the base. 7+7

6. Expand in Fourier series

$$f(x) = \begin{cases} -x & -4 \leq x \leq 0 \\ x & 0 \leq x \leq 4 \end{cases} \text{ and hence deduce that}$$

$$\frac{\pi^2}{6} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \dots \quad 14$$

7. (a) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2i - j - 2k$.

Find (i) $\vec{A} \times (\nabla \phi)$, (ii) $(\nabla \times \vec{A}) \times \vec{B}$, where

$$\vec{A} = x^2zi + yz^3j - 3xyzk$$

$$\vec{B} = 3xi + 4zj - xyzk \text{ and } \phi = xyz^2z \quad 7+7$$

8. Investigate for what values of a and b , the equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+az=b$ have (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions. 14

9. (a) Determine the rank and nullity of the following linear transformation :

$$T: V_4 \rightarrow V_3$$

defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 - x_2, x_2 + x_3, x_3 - x_4)$$

(7)

(b) Determine the eigenvalues and eigenvectors of the following matrix :

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

7+7

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