

Code : 211303

B.Tech 3rd Semester Exam., 2019 (Old Course)

MATHEMATICS—III

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. **1** is compulsory.

1. Choose the correct answer/Fill in the blank
(any seven) : $2 \times 7 = 14$

(a) $e^{-x}(c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x) + c_3 e^{3x}$ is the general solution of

(i) $d^3y/dx^3 + 4y = 0$

(ii) $d^3y/dx^3 - 8y = 0$

(iii) $d^3y/dx^3 + 8y = 0$

(iv) $d^3y/dx^3 - 2d^2y/dx^2 + dy/dx - 2 = 0$

(b) If $\int_{-1}^1 P_n(x) dx = 2$, then n is

(i) 2

(ii) 1

(iii) -1

(iv) None of the above

(c) If J_0 and J_1 are Bessel functions, then $J_1(x)$ is given by

(i) $-J_0$

(ii) $J_0(x) - 1/x J_1(x)$

(iii) $J_0(x) + 1/x J_1(x)$

(iv) None of the above

(d) The partial differential equation from $z = (c + x)^2 + y$ is

(i) $z = \left(\frac{\partial z}{\partial x}\right)^2 + y$

(ii) $z = \left|\frac{\partial z}{\partial y}\right|^2 + y$

(iii) $z = \frac{1}{4} \left(\frac{\partial z}{\partial x}\right)^2 + y$

(iv) $z = \frac{1}{4} \left(\frac{\partial z}{\partial y}\right)^2 + y$

(3)

(e) The particular integral of

$$(2D^2 - 3DD' + D'^2)z = e^{x+2y}$$

is

(i) $\frac{1}{2}e^{x+2y}$

(ii) $-\frac{1}{2}e^{x+2y}$

(iii) xe^{x+2y}

(iv) x^2e^{x+2y}

(f) The solution of $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, given $u(0, y) = 8e^{3y}$, is ____.

(g) If $f(z) = u(x, y) + iv(x, y)$ is analytic, then $f'(z)$ is equal to

(i) $\frac{\partial u}{\partial x} - i \frac{\partial v}{\partial x}$

(ii) $\frac{\partial u}{\partial x} - i \frac{\partial v}{\partial y}$

(iii) $\frac{\partial v}{\partial y} - i \frac{\partial v}{\partial x}$

(iv) None of the above

(4)

(h) The value of

$$\int_C (4x^3 dx + 3y^2 z^2 dy + 2y^3 z dz)$$

where C is any path joining A(-1, 1, 0) to B(1, 2, 1) is

(i) 0

(ii) 1

(iii) 8

(iv) -8

(i) The mean and variance of normal distribution

(i) are same

(ii) cannot be same

(iii) are sometimes equal

(iv) are equal in the limiting case, as $n \rightarrow \infty$

(j) In Poisson distribution, the second moment about the origin is 12. Then its third moment about mean is

(i) 2

(ii) 3

(iii) 5

(iv) 0

2. (a) Define an ordinary point and find the series solution of the differential equation

$$(x^2 - 2x + 1) \frac{d^2y}{dx^2} + (4x - 4) \frac{dy}{dx} + (x^2 - 2x + 3)y = 0$$

about the point $x=1$.

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- (b) Prove that

$$x^2 \frac{d^2}{dx^2} J_n(x) = (n^2 - x^2 - n) J_n(x) + x J_{n+1}(x), \\ n = 0, 1, 2, \dots \quad 4$$

3. (a) Derive the following recurrence formulae :

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$$(i) J'_n(x) = \frac{n}{x} [J_n(x) - J_{n+1}(x)]$$

$$(ii) J_{n+1}(x) = \frac{2n}{x} [J_n(x) - J_{n-1}(x)]$$

- (b) Show that

$$\int_{-1}^1 x^2 P_{n-1} P_{n+1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)} \quad 6$$

4. (a) Solve the equation

$$x(y^2 - z^2)p + y(z^2 - x^2)q - z(x^2 - y^2) = 0 \quad 8$$

- (b) Classify the following differential equation :

$$(1 - x^2) \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + (1 - y^2) \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + \\ 3x^2 y \frac{\partial z}{\partial y} - 2z = 0$$

5. The ends A and B of a rod of length 20 cm are at temperatures 30 °C and 80 °C until steady state prevails. Then the temperature of the rod ends are changed to 40 °C and 60 °C respectively. Find the temperature distribution function $u(x, t)$. The specific heat, density and the thermal conductivity of the material of the rod are such that the combination

$$\frac{k}{\rho \sigma} = c^2 = 1$$

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6. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied.

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- (b) If $f(z) = u + iv$ is an analytic function and $u + v = (x + y)(2 - 4xy + x^2 + y^2)$, then find u, v and analytic function $f(z)$.

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(7)

7. Evaluate the following :

$7 \times 2 = 14$

(a) $\int_C |z| dz$, where C is the left half of the unit circle $|z|=1$ from $z=-i$ to $z=i$

(b) $\int_C \frac{1}{z(z+\pi i)} dz$, where C is the circle $|z+3i|=1$

8. (a) Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx & , 0 \leq x < 1 \\ k & , 1 \leq x < 2 \\ -kx + 3k & , 2 \leq x < 3 \end{cases}$$

Then (i) determine k and find (ii) mean of X, (iii) variance of X and (iv) moment generating function of X.

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(b) Let X be a continuous random variable with moment generating function

$$M_X(t) = \cos t e^{5t + 9t^2}$$

Find mean and variance of X.

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9. (a) Define exponential distribution and find its mean and variance.

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(8)

(b) One of the major contributors to air pollution is hydrocarbon emitted from the exhaust system of automobiles. Let Y denote the numbers of grams of hydrocarbons emitted by an automobile per mile. Assume that Y has a log-normal distribution with parameters $\mu = 0.7$ and $\sigma = 0.2$. Find the probability that a randomly selected automobile will emit between 1.2 grams and 1.54 grams of hydrocarbons per mile. Given

$$F_z(-2.588) = 0.0048$$

$$\text{and } F_z(-1.341) = 0.0901$$

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